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A study on ranked bidirectional evolutionary structural optimization (R-BESO) method for fully stressed structure design based on displacement sensitivity

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Abstract

Evolutionary Structural Optimization(ESO) method is well known as one of several topology optimization methods and has been applied to a lot of optimization problems. While ESO method evolves the given model into an optimum by subtracting several elements, in AESO method elements are added in a previous step of the evolutionary procedure. And in BESO(Bidirectional ESO) method, some elements are either generated or eliminated from a previous model of evolutionary procedure. In this paper, Ranked Bidirectional Evolutionary Structural Optimization(R-BESO) method is introduced as one of the topology optimization methods using an evolutionary algorithm and is applied to several optimization problems. The method can get optimum topologies of the structures throughout fewer iterations comparing with previous several methods based on ESO. R-BESO method is similar to BESO method except that elements are generated near a candidate element according to the rank calculated by sensitivity analyses. The displacement sensitivity analysis was adopted by the nodal displacements of a candidate element in order to determine a rank on the free edges for two dimensional model or the free surfaces for three dimensional model. In this paper, R-BESO method is proposed as another useful design tool like the previous ESO and BESO method for the two bar frame problem, the Michell type structure problem and the three dimension short cantilever beam problem, which had been used to verify reasonability of ESO method family. For the three dimensions short cantilever beam problem an optimized topology could be obtained with much fewer iterations with respect to the results of other ESO methods.

Keywords: Ranked bidirection evolutionary structure optimization(R-BESO); Performance index(PI); Stress ratio(SR); Weight ratio(WR); Candidate element

1. Introduction

Generally the structural optimization aims at searching shapes or sizes of a structure under given loads and boundary conditions with minimum weight or minimum manufacturing cost while satisfying some constraints such as stresses, displacements, natural frequencies and buckling loads. In nature, the cases optimizing one's body may be found in growth and evolution of many kinds of lives. For instance, trees are growing up with gradually changing annual rings and the shape of their stems is being modified to get more sunshine and to make strongly support under given environment.

In designing structures, commercial FEA codes and optimization codes are already known as one of the useful tools. Although commercial optimization codes are powerful, a lot of field engineers cannot use them practically due to several efforts, for example cost or time, in order to apply to their problems, still now. And some engineers may have some troubles to

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use an optimization technique because the concept may be too complex for all engineers to understand easily and a lot of hours will be spent in applying to some real problems. So, most of the field engineers have searched for an available and easier optimization method extending a commercial FEM code which is used in a structure design presently.

The topology optimization is recently interesting for many engineers, which searches the first phase in structure design or an optimum connectivity of given components. Although the concept of the topology optimization proposed long time ago, the application has not improved significantly due to numerical difficulties. Size and shape optimization cannot search a structural topology because an initial topology has been maintained throughout an optimization procedure. In a structural design, a topology of structure should first be decided in order to satisfy various constraints such as load and boundary conditions. So the topology optimization method is one of the useful tools of the many structural designers who should determine a prospective feature of a new product.

Evolutionary Structural Optimization(ESO) method[1-8] has several benefits ; its concept is not difficult to understand and it can be easily expanded using commercial FE packages, respectively. The basic concept of ESO is that some material having less contribution degree to the given environments and criteria is removed from an initially defined FE model. For a stress problem, the materials involved in a lower stress in comparison with stress level in other regions are eliminated throughout evolutionary procedures, and so the contribution of finally remained materials becomes more equal over all the materials. In contrast to removing procedure, material addition procedure from the least material structure was called by added ESO(AESO). And Bidirectional ESO (BESO) [6] method was developed, which included both removal and additional procedures. BESO method cannot only get an optimal design throughout less iteration number with comparison with ESO but also prevent reaching local optimums. Reduction of iteration number means that the method may lead to saving of a computational cost. The methods based on ESO have been applied to various structure design problems of some stress analysis and modal analysis sides.

ESO method has been applied to elastic contact optimization problems using gap and beam elements [9]. The gap size of the gap elements was used as a design variable, and an optimal profile for each junction was eventually given. The computation in solving a contact problem generally requires more cost with respect to other linear problems because it is one of typical nonlinear problems. Kim et al[10] and Lee [11] studied on the reliability based topology optimization and robust design optimization.

Generally an optimized model through topology optimization procedures has non-smooth shapes and then some procedures in which outlines of the model are made continuous should be applied. Some researchers [12] had studied on optimizing topologies with other methods and generating CAM data in order to manufacture a structure based on the optimum.

Generally stresses caused by given load conditions may be reduced if a structure is made heavy. A heavy structure, however, may violate several design constraints in some fields and need more cost with respect to light one. Fully stressed structure means that all materials composed of a structure are uniformly contributed to several load conditions. So, local stress concentration may disappear over the structure under the given load conditions and the maximum stress can be reduced.

In this paper, R-BESO method is proposed as one of useful design tools in order to search an optimized structure topology. It is developed and extended out of BESO method using a sensitivity analysis for nodal displacement of elements. While elements are constantly generated on the existing element by criteria in BESO, a group of elements are differently added in accordance with calculated ranks by the sensitivity in R-BESO. For instance, the most number of elements are attached on the first rank side of which difference between nodal displacements for the existing element is the highest. Thus the number of structure analyses can be reduced over the optimization procedures significantly. And Performance Index(PI) defined by Stress Ratio(SR) and Weight Ratio(WR) is used as the objective function. We can get an optimum topology for several problems at lower iteration number of R-BESO comparing with that of BESO.

2. Performance index for fully stressed structure

A good structure in stress design side can be told that all of the materials are in an equal stress state as well as the weight is small. But stress concentration phenomena cannot be avoided for a real structure. A lot of engineers are eager to find a topology with lower stress concentration.

In this paper we want to get a topology of a fully stressed and light structure. So, Performance Index(PI) is defined using stress and weight factor as the objective function in the optimization procedure.

$$PI = \frac{1}{SR} \times \frac{1}{WR} \tag{1}$$

$$SR = \frac{(\sigma_{av})_c}{\sigma_{ref}} \tag{2}$$

$$WR = \frac{N}{N_{ref}} \tag{3}$$

PI consists of both a stress ratio and a weight ratio for a current FE model. The stress ratio is defined using an average Mises stress of candidate elements of each iteration and the reference stress which is an allowable design stress. If the stress ratio is smaller than 1, the present structure may be told that it is safe under the considered conditions. The weight ratio indicated the ratio between the number of elements for a current model and the reference number. The reference number is chosen with the number of all elements for an initial model. So, the lightest structure can be obtained with the smallest weight ratio. If a considered structure is made of the same material, the ratio between the numbers of elements for FE model is the same meaning as the weight ratio for that.

Consequently, FE models among evolutionary steps can be estimated by PI in order to choose an effective light structure in which most materials may contribute to supporting the given loads. With a viewpoint of optimization, maximizing PI can lead to minimizing stress owing to reducing the degree of stress concentration and minimizing the weight of a structure. Then we can obtain a fully stressed and light structure.

3. Ranked bidirectional evolutionary optimization(R-BESO)

ESO method searches an optimum topology from the maximum design domain throughout removing less efficient materials within the full domain. Thus a structure in the previous step has been heavier than the present one. On the other hand, AESO method is known as the procedure adding some materials to the minimum design domain around the region under higher stress. The structure grows through AESO procedure because a material is added on the only outside of structure's border.

BESO is classified into additional procedure and removal procedure and the volume history according to an optimization procedure is shown in Fig. 1. These results are from Querin's papers[5, 6] about the two bar frame and the Michell type structure, which have been generally used to compare with other methods of topology optimization. It is true that the weight of FE model during the evolutionary steps is constantly increased until 45 iteration steps and 61 iteration steps for each case. These amounts of the steps are 66% and 96% of the total iteration step number reaching an optimum. This says that if the rate of the growth of a structure was higher, then the total iteration number required to approach an optimal topology could be very effective on reducing the cost. In other words, if elements were fast generated around the region involving high stress, the iteration step number could be significantly shortened comparing with that of BESO.

Displacement sensitivity analysis was previously introduced to BESO in order to increase the structure volume in elements additional procedure. Only one element is added on each free edge of an element chosen due to the sensitivity analysis. In this study, however, several elements are generated on each free edge corresponding to the ranks determined by the sensitivity analysis. So, each free edge of a element has a rank which is an order of difference between nodal displacements. For instance, rank1 is the biggest displacement difference of two nodes for two dimensional model or four nodes for three dimensional model.

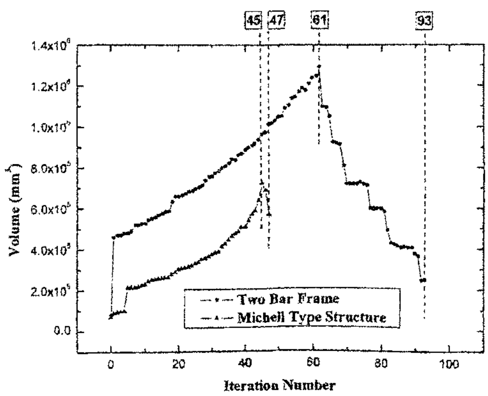


Fig. 1. Volume evolution history for the two bar frame and Michell type structure optimization problems using the BESO method [6].

As the same as other FE model of ESO family, considered elements are a square shape for two dimensional model and a cubic shape for three dimensional model. An element for two dimensional model and three dimensional model have 4 and 8 nodes, respectively. α_{ij}^{2D} indicates the displacement sensitivity for two dimensional model, and node i and j should be the same plane parallel to X-Z or Y-Z plane. α_{ij}^{3D}, the displacement sensitivity for three dimensional model, is absolute value of the difference between the maximum displacement and the minimum displacement of the 4 nodes which are posed onto the same plane parallel to X-Y, Y-Z or X-Z plane. And the rank is determined for each free edge of two dimensional model and for each free surface of three dimensional model.

$$\alpha_{ij}^{2D} = \left|u_i - u_j\right| \qquad i,j = \{1,\ldots,4\} \;\; ; \;\; i \neq j \tag{4}$$

$$\begin{aligned} \alpha_{ij}^{3D} &= \left|(u_i)_{\max} - (u_j)_{\min}\right| \qquad i,j = \{1,\ldots,8\} \\ &\text{where} \quad x_i = x_j \\ &\text{or} \quad y_i = y_j \\ &\text{or} \quad z_i = z_j \end{aligned} \tag{5}$$

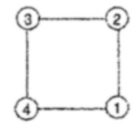
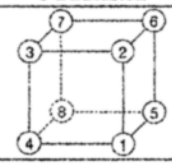
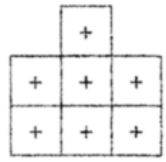
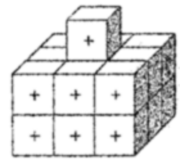
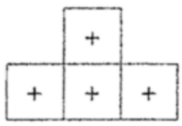
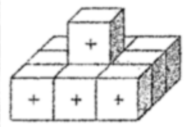
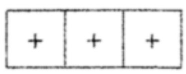
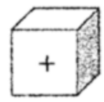
Table 1 denotes types of added elements corresponding to each rank for a free edge or a free surface for two or three dimensional model. For two dimensional model, stacked three elements are added on the free edge of rank 1 and four elements are created at both sides of them. And, three elements are also generated on a free surface determined by rank 1 for three dimensional model.

Actually, the number of added elements on a selected free surface for three dimensional model is the same as the number of added elements on a selected free edge for two dimensional model. To make the outline of the updated model smooth, several extra elements are generated on the sides of the elements added on free surface just like two dimension model.

Finally 7, 4 and 3 elements may be added to each free edge of a candidate element for two dimensional models, and 19, 10 and 1 element may be generated on each free surface for three dimensional models respectively. So, R-BESO can evolve a structure with faster growth rate comparing with BESO.

The mathematical representation of R-BESO is as follows

Table 1. Added element types corresponding to the rank onto free edges or free surfaces of a candidate element.

	2 Dimension	3 Dimension
Candidate Element		
Rank 1		
Rank 2		
Rank 3		

$$\text{Maximize}: \; f(\chi,\eta) = PI = \frac{1}{SR} \times \frac{1}{WR} \tag{6}$$

$$\text{Subject to}: \; \sum_{i=1}^{N} [Ku]_i - \{F\} = 0 \tag{7}$$

$$\begin{aligned} &\chi(\sigma_i - \sigma_L) \geq 0 \\ &\text{where} \quad \sigma_L = \sigma_{av} - RR\left(\sigma_{av} - \sigma_{\min}\right) \\ &\qquad \chi = \{0,1\} \end{aligned} \tag{8}$$

$$\begin{aligned} &\eta(\sigma_i - \sigma_U) \geq 0 \\ &\text{where} \quad \sigma_U = \sigma_{av} + IR\left(\sigma_{\max} - \sigma_{av}\right) \\ &\qquad \eta = \{0,3,4,7\} \quad \text{for} \;\; 2\,\text{dimension} \\ &\qquad \eta = \{0,1,10,19\} \quad \text{for} \;\; 3\,\text{dimension} \end{aligned} \tag{9}$$

where RR and IR indicate rejection ratio and inclusion ratio, respectively. They are ones of dominant factors in evolving a structure from an initial model to an optimum topology model. RR and IR are applied to determining a lower limit stress (σ_L) and an upper limit stress (σ_U) in evolutionary procedures, respectively. If an element stress involved in the element chosen of a present model is smaller than the lower limit stress or larger than the upper limit stress, then the element may be removed from a present model or some elements may be added around that according to the rank onto free edges. It may be said that the evolution is achieved throughout removal and additional procedure over all elements for a current model, briefly.

The procedure of R-BESO is summarized in Fig. 2 and composed of the fast evolution module and the normal evolution module. Elements are added using the previously decided rank and removed in the fast evolution module. In normal evolution module, the procedure deciding any ranks for free edges or free surfaces is excluded out of the fast module as shown in Fig. 3. The procedure that governs the R-BESO is as follows.

Step 1 : The maximum design domain must be specified, in which structure elements can be made throughout the optimization procedure, and it is necessary to define the positions under some required loads and boundary conditions.

Step 2 : The structure is meshed using square elements for a two dimensional model or cubic elements for a three dimensional model.

Step 3 : The initial model is generated using the elements connecting between loaded regions and bounded regions. This may be said by the smallest and simplest model which is satisfied with the considered condition and is made by removal of unselected elements from the design domain.

Step 4 : Choose IR(Inclusion Ratio), RR(Rejection Ratio) and ER(Evolution Ratio) which are ones of dominant factors of the optimization procedure.

Step 5 : Define some constraints

Step 6 : Carry out a finite element analysis for the initial model and obtain structure stresses of all elements and nodal displacement.

Step 7 : Start the fast evolution module

Step 8 : Calculate maximum, minimum and average of the stresses.

Step 9 : Select the candidate elements having any free edges or free surfaces.

Step 10 : Calculate the upper limit stress σ_U and the lower limit stress σ_L .

Step 11 : The rank on a free edge or a free surface of a candidate element is determined by nodal displacement sensitivity analysis and elements is generated corresponding to the rank. The elements having stresses lower than σ_L are removed out of a present model.

Step 12 : Conditions of an updated model obtained after both the element addition process and the element removal process, are examined using the evolution constraints defined previously. If they satisfy the

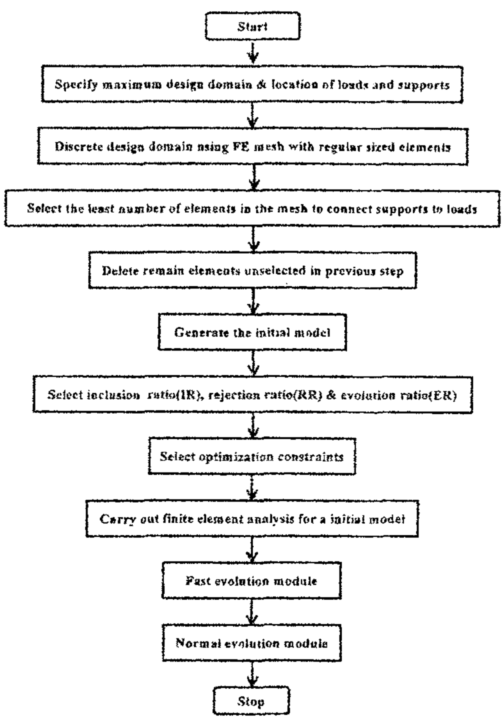


Fig. 2. Main flow chart for the R-BESO algorithm.

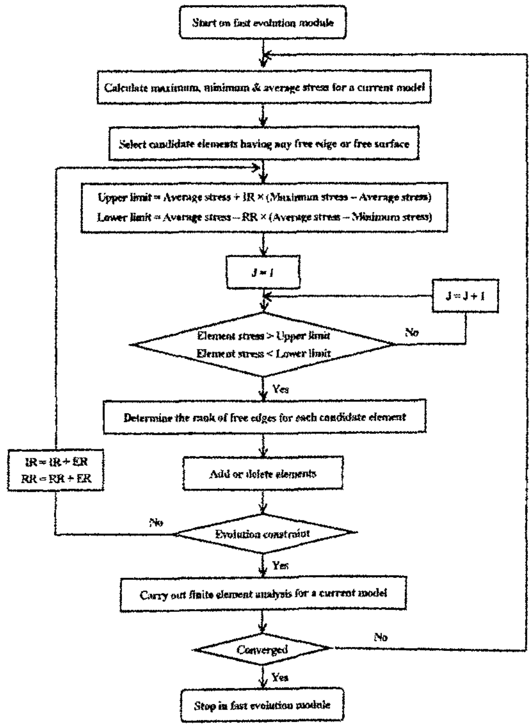


Fig. 3. Flow chart for the fast evolution module of the R-BESO.

constraints then go to next procedure. Otherwise IR and RR are updated using ER(Evolution Ratio).

$$IR_{new} = IR_{old} + ER$$
$$RR_{new} = RR_{old} + ER \tag{10}$$

Step 13 : After finite element analysis for an evolved model, calculate the performance index.

Step 14 : Steps from 8 to 13 are repeated until the performance index for an evolved model meets the following convergence conditions.

$$|PI_c - PI_{av}| - (\Delta PI)_{av} < 0 \tag{11}$$

where, $PI_{av} = \frac{\sum_{i=1}^{4} PI_{c-i}}{4}$

$$(\Delta PI)_{av} = \frac{\sum_{i=1}^{4} \Delta PI_i}{4} \tag{12}$$

where, $\Delta PI_i = PI_{c-i} - PI_{av} \qquad (i = 1, \cdots, 4)$

where, PI_c denotes the performance index for a present iteration model.

Step 15 : After the fast evolution module, start the normal evolution module excluding the sensitivity analysis procedure. So, elements are added onto all free edges or all free surfaces of a candidate element just corresponding to rank 3. To have higher accuracy rather than that for the fast module, the convergence criteria are changed as following.

$$PI_c - PI_{av} < 0 \tag{13}$$

where, $PI_{av} = \frac{\sum_{i=1}^{6} PI_{c-i}}{6}$

4. Results and discussion

To show some advantages of R-BESO method, two bar frame, Michell type structure and three dimensional short cantilever beam problems were used. They have been used in many studies for a topology optimization in order to be compared with other algorithms. In this study, the results for those problems by R-BESO method are compared with BESO results [6] in order to show the characteristics of R-BESO.

4.1 Two bar frame problem

Fig. 4 indicates the dimensions for the two bar frame problem with the maximum design domain and the mesh density for the finite element model. Individual plate element size and mesh density are equal to those used in the BESO study. The initial minimum structural domain means the first FE model from which evolutionary iterations are stated.

History for the objective function, PI, with processing iteration number is shown in Fig. 5. In fast evolution procedure, PI is sharply increased with respect to that in normal evolution procedure. PI is about 3.35 in the first step, and is increasing with processing itera tion steps. It largely continues to be raised until reach-

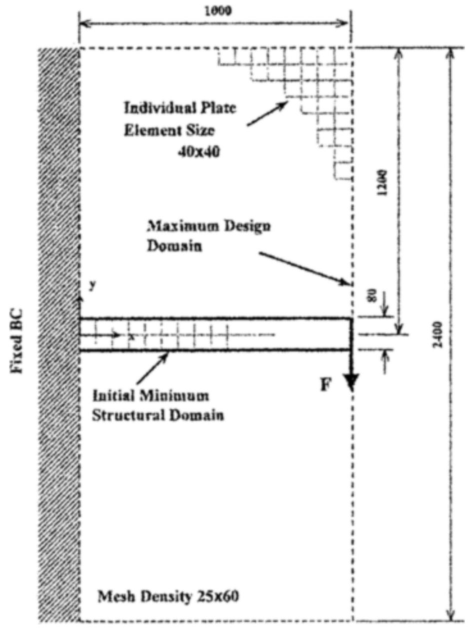


Fig. 4. Design domain for the two bar frame problem (unit: mm).

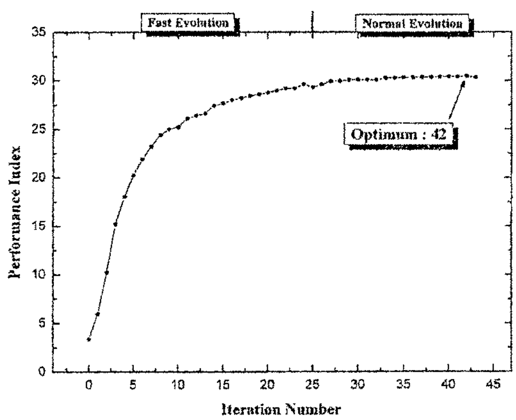
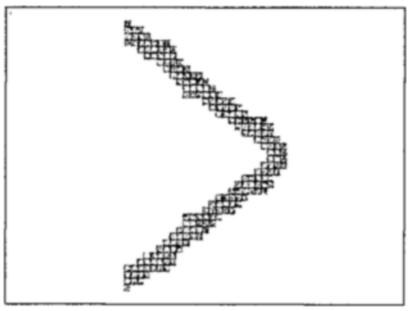


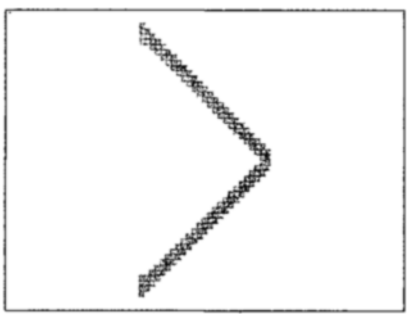
Fig. 5. Evolutio history of performance index for the two bar frame problem.

ing the maximum value, about 30.44, at iteration number 42. Deviations of PI in the normal evolution procedure are smaller than those in the fast evolution procedure. Iterations are finished when a current model satisfies the optimization conditions.

Optimal topologies through R-BESO and BESO are shown in Fig 6, respectively. R-BESO searched the optimum through 42 iterations, which was equivalent to 45% of the required iterations for BESO. R-BESO can be said a fantastic method in reducing computational costs. Two results are roughly similar to each other but the angle between two bars of BESO's result is the larger than that of R-BESO's result. This means that the distance between two supporting positions of the model by R-BESO is smaller than that by BESO. But the width of each bar of R-BESO's result is not even and is wider than BESO. Despite these differences between the two topologies, those are as just a few differences as may be ignored in many real situations because an optimized structure may not be taken by only a topology optimization tool. Particularly, it was known that saving costs and reduction in required hours are the most important issue in various industries. If described in a point of that side, R-BESO may be accepted in many kinds of industries. And even if using the structure topology obtained by R-BESO, some following processes, such as shape and size optimization, may be made eliminate the differences incurred in the previous process.



(a) R-BESO(Iteration number = 42)



(b) BESO[7] (Iteration number = 93)

Fig. 6. Optimal topologies for the two bar frame problem using the R-BESO and the BESO method.

Table 2 represents stresses and the total number of elements of obtained optimal topologies using each method. BESO can be used in order to obtain the lighter structure with 77% of the weight by R-BESO. The average stress for the R-BESO's model is smaller than that of the BESO's, but the maximum stress of the R-BESO's is the larger. For R-BESO, both of the difference between the maximum and minimum stress and the number of the model's elements are the larger than BESO's. And the ratio between the maximum stress and the average stress from R-BESO is the higher than BESO's. Stating PI for each result, which is defined in this paper, the value calculated from BESO is larger than R-BESO. However, the difference between them is 3.410 %, which may be very small in common engineering senses. So, we may briefly say that R-BESO's model is one of closely optimized models for a fully stressed structure.

4.2 Michell type structure problem

The Michell type structure has been said that both ends are under fixed boundary conditions and loads

Table 2. Comparison of optimal models for the two bar frame obtained by the R-BESO and BESO.

Method	σ_{max} (Mpa)	σ_{av} (Mpa)	σ_{min} (Mpa)	N	PI
R-BESO	6.656 $(\sigma_{max}/\sigma_{av}=2.83)$	2.354	0.054 $(\sigma_{min}/\sigma_{av}=0.02)$	196	30.343
BESO	6.412 $(\sigma_{max}/\sigma_{av}=2.16)$	2.971	2.008 $(\sigma_{min}/\sigma_{av}=0.68)$	150	31.414 (Diff. 3.410%)

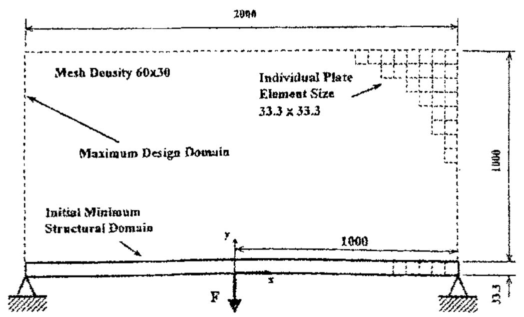


Fig. 7. Design domain for the Michell type structure problem (unit : mm).

are applied at middle point of the span as shown in Fig. 7. Maximum design domain, within which elements can be generated, has 1000 mm of the height.

Fig. 8 shows variations of PI in the evolution procedure for the Michell type structure using R-BESO. PI is steeply increased in the fast evolution procedure such as the trend of the two bar frame problem. The fast procedure is from the first to 8 iteration steps and PI is increased by 641% with respect to the initial model in the procedure. Finally an optimum topology is obtained at 22 step and totally calculated steps are required 25 iterations. In normal evolution procedure PI is increased with more stable than the fast procedure as for the two bar frame problem. Optimal topology of either R-BESO or BESO is shown in Fig. 9. For R-BESO, the optimized model requires just 47% computational costs in comparison with BESO. Reducing the costs is the best merit of R-BESO method. But the model has a few differences from the BESO's model.

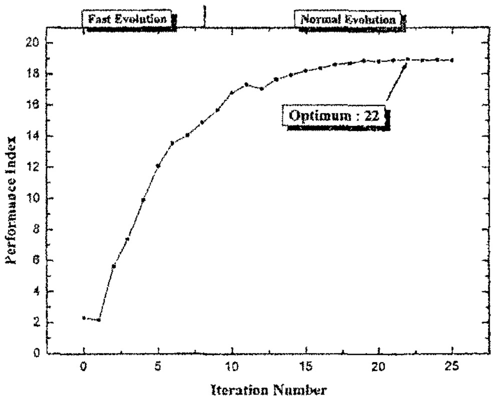
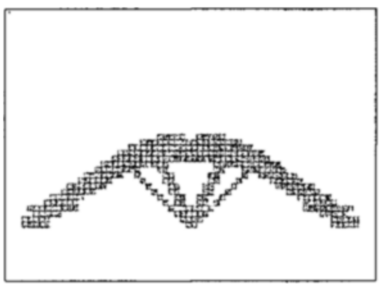
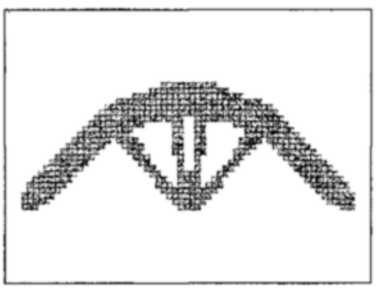


Fig. 8. Evolution history of performance index for the Michell type structure problem.



(a) R-BESO(Iteration number=22)



(b) BESO[7](Iteration number=47)

Fig. 9. Optimal topologies for the Michell type structure problem using the R-BESO and the BESO method.

In this case, the link of components is very similar to each other, although there are the differences of the width of each part and the height of the model from the two models.

Table 3 says that stress and the number of elements for the two topology optimized models. The number of elements for R-BESO is 65% with respect to BESO and then the lighter structure is obtained by R-BESO. But, the average stress of the optimized model for BESO has the smaller value. While the maximum stress is the larger, the ratio of the maximum stress to the average stress is the smaller for R-BESO.

This means that the contribution of material against the given load condition is evenly divided into all composed material, although the maximum stress is the higher. In the other side PI for BESO, which consists of both stress and weight factors, is the larger than R-BESO. The difference between two methods is calculated by 2.155%, which is smaller than the previous example. So, it is known by the objective function that the topology of R-BESO had nearly approached a fully stressed structure.

4.3 Three dimensional short cantilever beam problem

Although many cases of real engineering problems are in three dimensional state, simplified two dimen-

Table 3. Comparison of optimal models for the Michell type structure obtained by the R-BESO and BESO.

Method	σ_{max} (Mpa)	σ_{av} (Mpa)	σ_{min} (Mpa)	N	PI
R-BESO	10.676 $(\sigma_{max}/\sigma_{av}=3.93)$	2.719	0.042 $(\sigma_{min}/\sigma_{av}=0.02)$	326	18.953
BESO	8.591 $(\sigma_{max}/\sigma_{av}=5.01)$	1.714	0.184 $(\sigma_{min}/\sigma_{av}=0.11)$	506	19.370 (Diff. 2.155%)

sional models are applied for some problems in order to reduce some computational costs with the given accuracy limits. Other problems, which cannot be simplified under the given constraints, need to carry out a three dimensional analysis. A short cantilever beam problem without considerations of width effects is usually analyzed by means of a two dimensional plain strain problem. But a short cantilever beam having a narrow width may not be treated by a two dimensional problem and has to be analyzed using a three dimensional model.

Fig. 10 shows the design domain about the three dimensional short cantilever beam problem and identifies load and boundary conditions. The width of the beam is 40 mm smaller than the height. PI in the evolutionary procedure is shown in Fig. 11 and an opti mum topology is obtained at 12 iteration step. That is corresponding to about 5% and 4% of the iteration required for BESO and ESO, respectively. R-BESO may dramatically reduce computation costs for the same problem with respect to BESO and ESO. Considering that each step of the optimization procedure usually requires a lot of computational costs, R-BESO is much more effective for dimensional problems. Most of the generations of material in a current model are accomplished over the fast evolution procedure and in the normal evolution procedure PI approaches stable state.

Fig. 12 shows optimal topologies for R-BESO, ESO and BESO. There are some differences between ESO and BESO, and the iteration number spent by BESO is around 71% of that for ESO. But these topologies are largely similar to each other. The topology by R-BESO is heavier in the width than that for

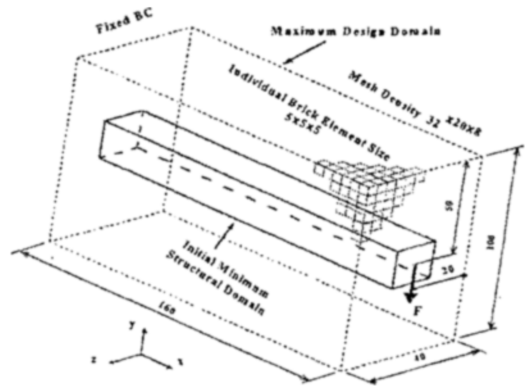


Fig. 10 Design domain for the 3D short cantilever beam problem (unit : mm).

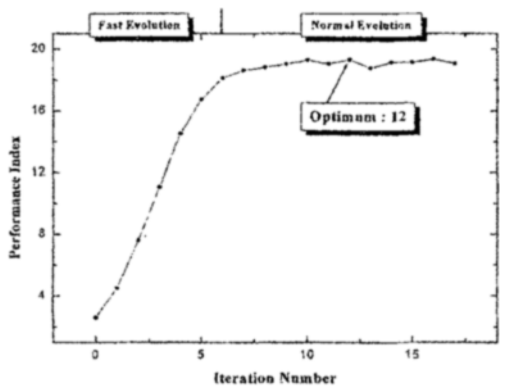
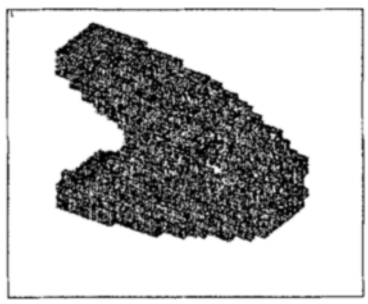
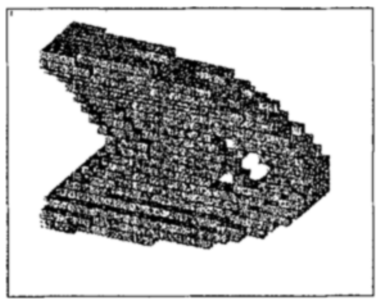


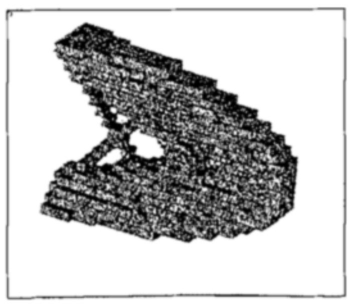
Fig. 11. Evolution history of performance index for the 3D short cantilever beam problem.



(a) R-BESO(Iteration number=12)



(b) ESO(Iteration number=317)



(c) BESO(Iteration number = 226)

Fig. 12. Optimal topologies for the 3D short cantilever beam problem using the R-BESO, ESO and BESO.

Table 4. Comparison of optimal models for the 3D cantilever beam obtained by the R-BESO, ESO and BESO.

Method	σ_{max} (Mpa)	σ_{av} (Mpa)	σ_{min} (Mpa)	N	PI
R-BESO	2.448 $(\sigma_{max}/\sigma_{av}=3.52)$	0.695	0.098 $(\sigma_{min}/\sigma_{av}=0.14)$	2671	19.306
ESO	3.134 $(\sigma_{max}/\sigma_{av}=2.74)$	1.142	0.359 $(\sigma_{min}/\sigma_{av}=0.31)$	1636	19.183 (Diff. 0.644%)
BESO	3.015 $(\sigma_{max}/\sigma_{av}=2.84)$	1.061	0.260 $(\sigma_{min}/\sigma_{av}=0.25)$	1762	19.171 (Diff. 0.707%)

the others but it is similar to that of the other methods.

The data of the obtained results are summarized in Table 4. The number of elements for R-BESO is calculated by 163% of ESO and 152% of BESO. The average stress for R-BESO is calculated by 61%, 66% for ESO and BESO, respectively. And PI for R-BESO is the smallest among the considered methods but the difference is smaller than 1%. Using the objective function, the most optimized topology could not be selected among these topologies. So, a topology optimization requires another consecutive procedure such as either shape or size optimization methods.

5. Concluding remarks

BESO method is one of the improved tools using ESO concept and has been told that the method is more effective than a typical ESO. BESO consists of the two procedures, adding and removing some materials. In BESO, the major portion of a structural evolution is occupied by the generation procedure in which elements are added to a previous model. For instances, two bar frame and Michell type structure problems spend more than half of the total computation costs to get an optimum.

R-BESO method had been developed in order to reduce the computational costs spent in the generation procedure. So, an optimum topology may be obtained much more economically because the costs are mainly used for the added procedure for BESO. In R-BESO, a group of material is added based on the rank determined by displacement sensitivity in evolutionary procedures. Added material groups are three types, 7, 4 and 3 elements for two dimension model and 19, 10 and 1 element for three dimension model.

To comparison with BESO, R-BESO was applied to several problems which were the same problems as those used in the BESO's study. For two dimension problems such as two bar frame and Michell type structure, R-BESO can search an optimum topology throughout 46% or 47% iteration of BESO's results, respectively. But the optimum topology obtained for each problem has a few differences from BESO results. The BESO's topologies are more optimized than those of R-BESO compared by the objective function. However, the different value of the objective function is smaller than 5% and considering another consecutive design process the differences may not affect a final product. The reduction of computational costs for R-BESO is fantastic for a three dimension short cantilever problem. The rate of iteration number required to reach an optimum topology for R-BESO to that for BESO is 5%. And the value of the objective function for R-BESO is a little lager than that for BESO. Namely, R-BESO's topology is the more optimized model than BESO's but the difference is very small.

In conclusion, R-BESO method may not approach the exact optimized topology but can make us obtain a closely optimized model which may has few differences for a final product. R-BESO can obtain an optimized topology throughout much smaller iterations comparison with BESO. Particularly, the reduction in the computational costs for three dimensional problems is dramatic so, it can be expected to be efficiently used in many areas demanding faster results.

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